Base *e* and Natural Logarithms

e is referred to as the natural base, or the Euler Number. An exponential function with base *e* is called a natural base exponential function.

The inverse of a natural base exponential function is called the natural log.

$\log_e x = \ln x$, or for example $\ln 4 = x \rightarrow \log_e 4 = x \rightarrow e^x = 4$

Write an equivalent exponential or logarithmic function.

$\ln e^x = \ln 8$

$x = \ln 8$

$\ln e^5 = \ln x$

$5 = \ln x$

$e^{\ln 25} = e^x$

$25 = e^x$

Write the expression as a single logarithm.

$3 \ln 10 - \ln 8$

$\ln 10^3 - \ln 8$

$\ln\left(\frac{1000}{8}\right)$

$= \ln 125$

$2 \ln 5 + 4 \ln 2 + \ln 5y$

$\ln 5^2 + \ln 2^4 + \ln 5y$

$\ln(25 \cdot 16 \cdot 5y)$

$\ln 2000y$

Solve each equation or inequality.

$4e^{-2x} - 5 = 3$

$+5 \quad +5$

$\frac{4e^{-2x}}{4} = \frac{8}{4}$

$\ln e^{-2x} = \ln 2$

$\frac{-2x}{-2} = \frac{\ln 2}{-2}$

$x = -\frac{1}{2}\ln 2$

$-\frac{\ln 2}{2}$

$3e^{4x} - 12 = 15$

$+12 \quad +12$

$\frac{3e^{4x}}{3} = \frac{27}{3}$

$\ln e^{4x} = \ln 9$

$\frac{4x}{4} = \frac{\ln 9}{4}$

$x = \frac{\ln 9}{4}$

$\frac{3 \ln 4x}{3} = \frac{24}{3}$

$e^{\ln 4x} = e^8$

$\frac{4x}{4} = \frac{e^8}{4}$

$x = \frac{e^8}{4}$

$\frac{5 \ln 6x}{5} > \frac{8}{5}$

$e^{\ln 6x} > e^{\frac{8}{5}}$

$\frac{6x}{6} > \frac{e^{\frac{8}{5}}}{6}$

$x > \frac{e^{\frac{8}{5}}}{6}$

Continuously Compounded Interest:

When interest is compounded continuously, the amount A in an account after t years is given by the formula:

$$A = Pe^{rt}$$

Where P is the principal and r is the annual interest rate expressed as a decimal.

When Angelia was born, her grandparents deposited $3000 into a college savings account paying 4% interest compounded continuously.

- Assuming there are no deposits or withdrawls from the account, what will the balance be after 10 years?

$A = 3000e^{(.04)(10)}$

$= 3000e^{.4}$

$A = \$4475.47$

- How long will it take the balance to reach at least $10,000?

$\frac{10000}{3000} < \frac{3000e^{.04t}}{3000}$

$\ln\frac{10}{3} < \ln e^{.04t}$

$\frac{\ln\frac{10}{3}}{.04} < \frac{.04t}{.04}$

$t > 30.099$

Approx 30 yrs

- If her grandparents want Angelia to have $10,000 after 18 years, how much would they need to invest?

$10000 = Pe^{(.04)(18)}$

$\frac{10000}{e^{.72}} = \frac{Pe^{.72}}{e^{.72}}$

$P = \$4867.52$