

Compound Inequalities

Solve each compound inequality and graph its solution.

1) $x - 4 \geq -8$ and $x - 9 \leq -4$

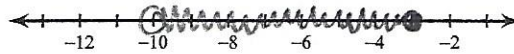


$$\begin{array}{r} x - 4 \geq -8 \quad \& \quad x - 9 \leq -4 \\ +4 \quad +4 \quad \quad \quad +9 \quad +9 \end{array}$$

$$\boxed{x \geq -4 \text{ and } x \leq 5}$$

$$\boxed{-4 \leq x \leq 5}$$

2) $-8p < 80$ and $p - 7 \leq -10$

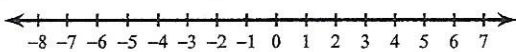


$$\begin{array}{r} -8p < 80 \quad \& \quad p - 7 \leq -10 \\ -8 \quad -8 \quad \quad \quad +7 \quad +7 \end{array}$$

$$\boxed{p > -10 \text{ and } p \leq -3}$$

$$\boxed{-10 < p \leq -3}$$

3) $-2 < 8x - 10 \leq -50$



$$\begin{array}{r} -2 < 8x - 10 \leq -50 \\ +10 \quad +10 \quad +10 \end{array}$$

$$\frac{8}{8} < \frac{8x}{8} \leq \frac{-40}{8}$$

$$1 < x \leq -5$$

NS

4) $-5 + 8a > 35$ and $2 - 5a > -38$



$$\begin{array}{r} -5 + 8a > 35 \quad \& \quad 2 - 5a > -38 \\ +5 \quad +5 \quad \quad \quad -2 \quad -2 \end{array}$$

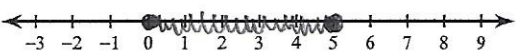
$$\frac{8a}{8} > \frac{40}{8}$$

$$\frac{-5a}{-5} > \frac{-40}{-5}$$

$$\boxed{a > 5 \text{ and } a < 8}$$

$$\boxed{5 < a < 8}$$

5) $7 + 2x \leq 17$ and $4x - 5 \geq -5$



$$\begin{array}{r} 7 + 2x \leq 17 \quad \& \quad 4x - 5 \geq -5 \\ -7 \quad -7 \quad \quad \quad +5 \quad +5 \end{array}$$

$$\frac{2x}{2} \leq \frac{10}{2}$$

$$\frac{4x}{4} \geq \frac{0}{4}$$

$$\boxed{x \leq 5 \text{ and } x \geq 0}$$

$$\boxed{0 \leq x \leq 5}$$

6) $n - 8 \leq 7 - 4n \leq n + 2$



$$\begin{array}{r} n - 8 \leq 7 - 4n \leq n + 2 \\ -n \quad -n \quad -n \end{array}$$

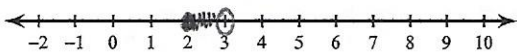
$$\begin{array}{r} -8 \leq 7 - 5n \leq 2 \\ -7 \quad -7 \quad \quad \quad -7 \end{array}$$

$$\frac{-15}{-5} \leq \frac{-5n}{-5} \leq \frac{-5}{-5}$$

$$3 \geq n \geq 1$$

$$\boxed{1 \leq n \leq 3}$$

7) $2p + 9 > 8p - 9 \geq 9 - p$



$$2p + 9 > 8p - 9 \geq 9 - p$$

$$\begin{array}{r} 2p + 9 > 8p - 9 \\ -2p \quad -2p \end{array}$$

$$\begin{array}{r} 8p - 9 \geq 9 - p \\ +p \quad +p \end{array}$$

$$\frac{9}{+9} > \frac{6p - 9}{+9}$$

$$\frac{9p - 9}{+9} \geq \frac{9}{+9}$$

$$\frac{18}{9} > \frac{6p}{9}$$

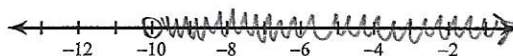
$$\frac{9p}{9} \geq \frac{18}{9}$$

$$3 > p$$

$$p \geq 2$$

$$\boxed{2 \leq p < 3}$$

8) $-4a < 12$ or $\frac{a}{5} > -2$



$$\frac{-4a}{-4} < \frac{12}{-4}$$

$$5 \cdot \frac{a}{5} > -2 \cdot 5$$

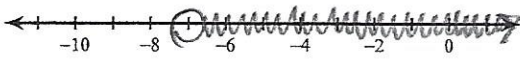
$$a > -3$$

$$a > -10$$

$$\boxed{a > -10 \text{ OR } a > 3}$$

$$\boxed{a > -10}$$

$$9) \frac{x}{7} > -1 \text{ or } \frac{x}{2} > 5$$



$$\frac{x}{7} > -1$$

$$x > -7$$

$$\frac{x}{2} > 5$$

$$x > 10$$

$$x > -7 \text{ OR } x > 10$$

$$x > -7$$

$$10) 10x - 3 \geq -3 \text{ or } 9x + 4 < -50$$



$$10x - 3 \geq -3$$

$$\frac{10x}{10} \geq \frac{0}{10}$$

$$x \geq 0$$

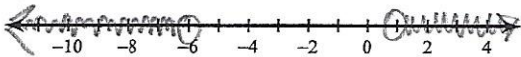
$$9x + 4 < -50$$

$$\frac{9x}{9} < \frac{-54}{9}$$

$$x < -6$$

$$x \geq 0 \text{ OR } x < -6$$

$$11) 5 + 8x > 13 \text{ or } 2 + 8x < -46$$



$$5 + 8x > 13$$

$$\frac{8x}{8} > \frac{8}{8}$$

$$x > 1$$

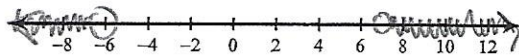
$$2 + 8x < -46$$

$$\frac{8x}{8} < \frac{-48}{8}$$

$$x < -6$$

$$x > 1 \text{ OR } x < -6$$

$$12) 8x + 1 < -47 \text{ or } -8 + 6x > 34$$



$$8x + 1 < -47$$

$$\frac{8x}{8} < \frac{-48}{8}$$

$$x < -6$$

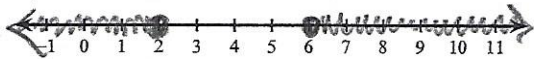
$$-8 + 6x > 34$$

$$\frac{6x}{6} > \frac{42}{6}$$

$$x > 7$$

$$x < -6 \text{ OR } x > 7$$

$$13) 4 - 2n \leq 10 - 5n \text{ or } 4n - 3 \leq 5n - 9$$



$$4 - 2n \leq 10 - 5n$$

$$4 + 3n \leq 10$$

$$\frac{3n}{3} \leq \frac{6}{3}$$

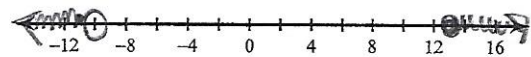
$$4n - 3 \leq 5n - 9$$

$$-3 \leq n - 9$$

$$6 \leq n$$

$$n \leq 2 \text{ OR } n \geq 6$$

$$14) 3 - 9p < -10p - 7 \text{ or } 5p - 3 \geq 10 + 4p$$



$$3 - 9p < -10p - 7$$

$$3 + p < -7$$

$$p < -10$$

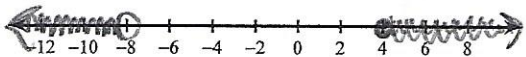
$$5p - 3 \geq 10 + 4p$$

$$p - 3 \geq 10$$

$$p \geq 13$$

$$p < -10 \text{ OR } p \geq 13$$

$$15) -5 + 9b \geq 31 \text{ or } 5b - 4 < -44$$



$$-5 + 9b \geq 31$$

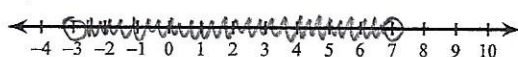
$$\frac{9b}{9} \geq \frac{36}{9}$$

$$b \geq 4 \text{ OR } b < -8$$

$$5b - 4 < -44$$

$$\frac{5b}{5} < \frac{-40}{5}$$

$$16) -9 < 5 - 2a < 11$$



$$-9 < 5 - 2a < 11$$

$$\frac{-14}{-2} < \frac{-2a}{-2} < \frac{6}{-2}$$

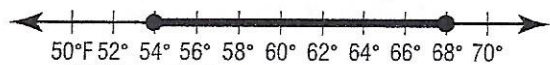
$$7 > a > -3$$

$$-3 < a < 7$$

3-2 Word Problem Practice

Solving Compound Inequalities

1. WEATHER Ken saw this graph in the newspaper weather forecast. It shows the predicted temperature range for the following day. Write an inequality to represent the number line graph.

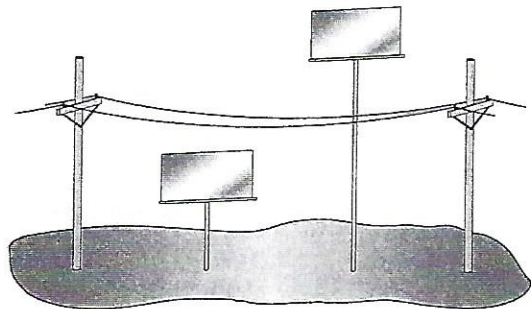


$$54 \leq x \leq 68$$

2. POOLS The pH of a person's eyes is 7.2. Therefore, the ideal pH for the water in a swimming pool is between 7.0 and 7.6. Write a compound inequality to represent pH levels that could cause physical discomfort to a person's eyes.

$$x < 7.0 \text{ OR } x > 7.6$$

3. STORE SIGNS In Randy's town, all stand-alone store front signs must be exactly 8 feet high. When mounted atop a pole, the combined height of the sign and pole must be less than 20 feet or greater than 35 feet so that they do not interfere with the power and phone lines. Write a compound inequality to represent the possible above-ground height of the poles.



$$p < 20 \text{ OR } p > 35$$

4. HEALTH The human heart circulates from 770,000 to 1,600,000 gallons of blood through a person's body every year. How many gallons of blood does the heart circulate through the body in one day?

$$\frac{770000}{365} \leq \frac{365g}{365} \leq \frac{1600000}{365}$$

$$2109.6 \leq g \leq 4383.6$$

5. HEALTH Body mass index (BMI) is a measure of weight status. The BMI of a person over 20 years old is calculated using the following formula.

$$BMI = 703 \times \frac{\text{weight in pounds}}{(\text{height in inches})^2}$$

The table below shows the meaning of different BMI measures.

BMI	Weight Status
less than 18.5	underweight
18.5 – 24.9	normal
25 – 29.9	overweight
more than 30	obese

Source: Centers for Disease Control

a. Write a compound inequality to represent the normal BMI range.

$$18.5 \leq W \leq 24.9$$

b. Write a compound inequality to represent the weight of an adult who is 6 feet tall that is within the normal BMI range.

$$W \cdot B = 703 \cdot \frac{W}{(h)^2} \cdot h^2$$

$$\frac{h^2 \cdot B}{703} = \frac{703W}{703}$$

$$W = \frac{Bh^2}{703}$$

$$W = \frac{(18.5)(72)^2}{703} = 136.4$$

$$W = \frac{(24.9)(72)^2}{703} = 183.6$$

$$136.4 \leq W \leq 183.6$$