Algebra 2

Name Key

7.5 Properties of Logs

Date _____ Hour ____

Expand each logarithm.

1) $\log_3 \sqrt[3]{a \cdot b \cdot c}$

$\log_3 (abc)^{\frac{1}{3}}$

$= \frac{1}{3}\log_3 (abc)$

$= \boxed{\frac{1}{3}\log_3 a + \frac{1}{3}\log_3 b + \frac{1}{3}\log_3 c}$

2) $\log_2 \left(\frac{u^3}{v}\right)^5$

$\log_2 \frac{u^{15}}{v^5}$

$= \log_2 u^{15} - \log_2 v^5$

$= \boxed{15\log_2 u - 5\log_2 v}$

3) $\log_5 \left(\frac{a}{b^2_3}\right)^3$

$\log_5 \frac{a^3}{b^6}$

$= \log_5 a^3 - \log_3 b^6$

$= \boxed{3\log_5 a - 6\log_3 b}$

4) $\log_5 \left(\frac{x}{y^5}\right)^6$

$\log_5 \frac{x^6}{y^{30}}$

$= \log_5 x^6 - \log_5 y^{30}$

$= \boxed{6\log_5 x - 30\log_5 y}$

5) $\log_3 (x^6 y^2)$

$\log_3 x^6 + \log_3 y^2$

$= \boxed{6\log_3 x + 2\log_3 y}$

6) $\log_3 (a^5 \cdot b)^2$

$\log_3 (a^{10} b^2)$

$= \log_3 a^{10} + \log_3 b^2$

$= \boxed{10\log_3 a + 2\log_3 b}$

7) $\log_3 (x^5 \cdot y)^4$

$\log_3 (x^{20} y^4)$

$= \log_3 x^{20} + \log_3 y^4$

$= \boxed{20\log_3 x + 4\log_3 y}$

8) $\ln (x^2 \cdot y)^5$

$\ln (x^{10} y^5)$

$= \ln x^{10} + \ln y^5$

$= \boxed{10\ln x + 5\ln y}$

Condense each expression to a single logarithm.

9) $4\log_5 w + \frac{\log_5 u}{3}$

$\log_5 w^4 + \frac{1}{3}\log_5 u$

$= \log_5 w^4 + \log_5 u^{\frac{1}{3}}$

$= \boxed{\log_5 (w^4 u^{\frac{1}{3}})}$

10) $5\log_3 x - 25\log_3 y$

$\log_3 x^5 - \log_3 y^{25}$

$= \boxed{\log_3 \frac{x^5}{y^{25}}}$

11) $\log_5 c + \dfrac{\log_5 a}{3} + \dfrac{\log_5 b}{3}$

$\log_5 c + \log_5 a^{\frac{1}{3}} + \log_5 b^{\frac{1}{3}}$

$= \log_5 (c a^{\frac{1}{3}} b^{\frac{1}{3}})$

12) $5\log_4 u - 6\log_4 v$

$\log_4 u^5 - \log_4 v^6$

$= \log_4 \dfrac{u^5}{v^6}$

13) $3\log_2 w + \dfrac{\log_2 u}{2}$

$\log_2 w^3 + \log_2 u^{\frac{1}{2}}$

$= \log_2 (w^3 u^{\frac{1}{2}})$

14) $3\log_9 u + 9\log_9 v$

$\log_9 u^3 + \log_9 v^9$

$= \log_9 (u^3 v^9)$

15) $\log_8 a + \log_8 b + 3\log_8 c$

$\log_8 a + \log_8 b + \log_8 c^3$

$= \log_8 (abc^3)$

16) $20\log_2 x - 4\log_2 y$

$\log_2 x^{20} - \log_2 y^4$

$= \log_2 \dfrac{x^{20}}{y^4}$

Use the properties of logarithms and the logarithms provided to rewrite each logarithm in terms of the variables given.

17) $\log_8 7 = X$
$\log_8 6 = Y$
$\log_8 9 = Z$
Find $\log_8 81$

$\log_8 9^2$

$= 2\log_8 9$

$= 2Z$

18) $\log_8 6 = A$
$\log_8 10 = B$
$\log_8 9 = C$
Find $\log_8 \dfrac{1}{6}$

$\log_8 6^{-1}$

$= -1\log_8 6$

$= -1A$

19) $\log_9 7 = X$
$\log_9 10 = Y$
$\log_9 6 = Z$
Find $\log_9 700$

$\log_9 (7 \cdot 10^2)$

$= \log_9 7 + \log_9 10^2$

$= \log_9 7 + 2\log_9 10$

$= X + 2Y$

20) $\log_7 8 = P$
$\log_7 10 = Q$
$\log_7 9 = R$
Find $\log_7 \dfrac{49}{9}$

$\log_7 49 - \log_7 9$

$= \log_7 7^2 - \log_7 9$

$= 2\log_7 7 - \log_7 9$

$= 2 - R$

p489 36-58, 65

Key

(36) $\log_3 6 + \log_3 x = \log_3 12$

$\log_3 6x = \log_3 12$

$\frac{6x}{6} = \frac{12}{6}$

$\boxed{x = 2}$

(37) $\log_4 a + \log_4 8 = \log_4 24$

$\log_4 8a = \log_4 24$

$\frac{8a}{8} = \frac{24}{8}$

$\boxed{a = 3}$

(38) $\log_{10} 18 - \log_{10} 3x = \log_{10} 2$

$\log_{10} \frac{18}{3x} = \log_{10} 2$

$3x \cdot \frac{18}{3x} = 2 \cdot 3x$

$\frac{18}{6} = \frac{6x}{6}$

$\boxed{x = 3}$

(39) $\log_7 100 - \log_7 (y+5) = \log_7 10$

$\log_7 \frac{100}{y+5} = \log_7 10$

$(y+5) \cdot \frac{100}{y+5} = 10(y+5)$

$100 = 10y + 50$

$-50 \quad -50$

$\frac{50}{10} = \frac{10y}{10}$

$\boxed{y = 5}$

(40) $\log_2 n = \frac{1}{3}\log_2 27 + \log_2 36$

$\log_2 n = \log_2 27^{\frac{1}{3}} + \log_2 36$

$\log_2 n = \log_2 (27^{\frac{1}{3}} \cdot 36)$

$n = 27^{\frac{1}{3}} \cdot 36$

$n = 3 \cdot 36$

$\boxed{n = 108}$

(41) $3\log_{10} 8 - \frac{1}{2}\log_{10} 36 = \log_{10} x$

$\log_{10} 8^3 - \log_{10} 36^{\frac{1}{2}} = \log_{10} x$

$\log_{10} \frac{8^3}{36^{\frac{1}{2}}} = \log_{10} x$

$\frac{8^3}{36^{\frac{1}{2}}} = x$

$\frac{512}{6} = x$

$\boxed{x = \frac{256}{3}}$

(42) $\log_a 6n - 3\log_a x = \log_a x$

$\log_a 6n - \log_a x^3 = \log_a x$

$\log_a \frac{6n}{x^3} = \log_a x$

$\frac{6n}{x^3} = x \cdot x^3$

$6n = x^4$

$n = \frac{x^4}{6}$

(43) $2\log_b 16 + 6\log_b n = \log_b (x-2)$

$\log_b 16^2 + \log_b n^6 = \log_b (x-2)$

$\log_b (16^2 \cdot n^6) = \log_b (x-2)$

$\frac{256n^6}{256} = \frac{x-2}{256}$

$(n^6)^{\frac{1}{6}} = \left(\frac{x-2}{256}\right)^{\frac{1}{6}}$

$n = \left(\frac{x-2}{256}\right)^{\frac{1}{6}}$

(44) $\log_{10} z + \log_{10}(z+9) = 1$

$\log_{10} z(z+9) = 10^1$

$z(z+9) = 10$

$z^2 + 9z = 10$

$z^2 + 9z - 10 = 0$

$(z+10)(z-1) = 0$

$z = -10, 1$

$z = 1$

$\log_{10} 10 + \log_{10} \frac{1}{10} = 1$

$\log_{10} 1 + \log_{10}(10) = 1$

$0 + 1 = 1$ ✓

(45) $\log_3(a^2+3) + \log_3 3 = 3$

$\log_3 (a^2+3)3 = 3^3$

$(a^2+3)3 = 27$

$3a^2 + 9 = 27$

$-9 \quad -9$

$\frac{3a^2}{3} = \frac{18}{3}$

$\sqrt{a^2} = \sqrt{6}$

$a = \pm\sqrt{6}$

$\log_3((\sqrt{6})^2+3) + \log_3 3 = 3$

$\log_3 9 + 1 = 3$

$2 + 1 = 3$

(46) $\log_2(15b-15) - \log_2(-b^2+1) = 1$

$\log_2 \frac{15b-15}{-b^2+1} = \log_2 2^1$

$(-b^2+1)\frac{15b-15}{-b^2+1} = 2(-b^2+1)$

$15b - 15 = -2b^2 + 2$

$2b^2 + 15b - 17 = 0$

$2b^2 - 2b + 17b - 17 = 0$

$2b(b-1) \quad 17(b-1)$

$(2b+17)(b-1) = 0$

$b = -\frac{17}{2}, 1$

No IR Solutions

(47) $\log_4(2y+2) - \log_4(y-2) = 1$

$\log_4 \frac{2y+2}{y-2} = \log_4 4^1$

$(y-2)\frac{2y+2}{y-2} = 4(y-2)$

$2y + 2 = 4y - 8$

$-2y + 2 = -8$

$-2y = -10$

$y = 5$

$\log_4 \frac{10+2}{3} = 1$

$\log_4 4 = 1$ ✓

(48) $\log_6 .1 + 2\log_6 x = \log_6 2 + \log_6 5$

$\log_6 (\frac{1}{10} \cdot 100) = \log_6 10$

$\log_6 10 = \log_6 10$ ✓

$\log_6 (.1x^2) = \log_6 (2 \cdot 5)$

$.1x^2 = 10$

$10 \cdot \frac{1}{10} x^2 = 10 \cdot 10$

$\sqrt{x^2} = \sqrt{100}$

$x = \pm 10$

$\boxed{x = 10}$

(49) $\log_7 64 - \log_7 \frac{8}{3} + \log_7 2 = \log_7 4p$

$\log_7 (\frac{64}{\frac{8}{3}} \cdot 2) = \log_7 4p$

$64 \cdot \frac{3}{8} \cdot 2$

$\log_7 48 = \log_7 4p$

$\log_7 48 = \log_7 4^{1.2}$

48 ✓

$\frac{48}{4} = \frac{4p}{4}$

$\boxed{p = 12}$

(50) (a) $P = 5000(1 - .04)^t$

$\frac{P}{5000} = \frac{5000(1-.04)^t}{5000}$

$\frac{P}{5000} = (1 - .04)^t$

$\frac{P}{5000} = (.96)^t$

$\log_{.96} (\frac{P}{5000}) = t$

(b) 40 years

(51) $\log_8 (x-3) = \log_8 x - \log_8 3$

$\log_8 (x-3) = \log_8 \frac{x}{3}$ False

(52) $\log_5 22x = \log_5 22 + \log_5 x$

$\log_5 22x = \log_5 22x$

True

(53) $\log_{10} 19k = 19 \log_{10} k$

$\log_{10} 19k = \log_{10} k^{19}$

False

(54) $\log_2 y^5 = 5 \log_2 y$

$\log_2 y^5 = \log_2 y^5$

True

(55) $\log_7 \frac{x}{3} = \log_7 x - \log_7 3$

$\log_7 \frac{x}{3} = \log_7 \frac{x}{3}$

True

(56) $\log_4 (z+2) = \log_4 z + \log_4 2$

$\log_4 (z+2) = \log_4 (2z)$

False

(57) $\log_8 p^4 = (\log_8 p)^4$

False

(58) $\log_a \frac{x^2y^3}{z^4} = 2\log_a x + 3\log_a y - 4\log_a z$

$\log_a \frac{x^2y^3}{z^4} = \log_a x^2 + \log_a y^3 - \log_a z^4$

$\log_a \frac{x^2y^3}{z^4} = \log_a \frac{x^2y^3}{z^4}$

True

(65) $\log_b 24 = \log_b 20 + \log_b 4$

Algebra 2

Name Key

7.5 Extra Practice

Date _____ Hour ____

Expand each logarithm.

1) $\log_3 \left(\frac{x}{zy^2}\right)^6$

$6\log_3\left(\frac{x}{zy^2}\right)$

$= 6\log_3 x - 6\log_3 z + 6\log_3 y^2$

$= \boxed{6\log_3 x - 6\log_3 z + 12\log_3 y}$

2) $\log_8 (xy^2 \cdot z)^4$

$4\log_8(xy^2z)$

$= \boxed{4\log_8 x + 8\log_8 y + 4\log_8 z}$

3) $\log_4 (bc^5\sqrt{a})$

$\log_4(bc^5a^{\frac{1}{2}})$

$= \boxed{\log_4 b + 5\log_4 c + \frac{1}{2}\log_4 a}$

4) $\log_5 (w^4\sqrt[3]{u \cdot v})$

$\log_5(w^4(uv)^{\frac{1}{3}})$

$= \boxed{4\log_5 w + \frac{1}{3}\log_5 u + \frac{1}{3}\log_5 v}$

Condense each expression to a single logarithm.

5) $\ln w + 2\ln u + 5\ln v$

$\ln w + \ln u^2 + \ln v^5$

$= \boxed{\ln(wu^2v^5)}$

6) $\log_3 y + 6\log_3 z + \frac{\log_3 x}{2}$

$\log_3 y + \log_3 z^6 + \frac{1}{2}\log_3 x$

$= \log_3 y + \log_3 z^6 = \log_3 x^{\frac{1}{2}}$

$= \boxed{\log_3(yz^6x^{\frac{1}{2}})}$

7) $\log_4 x + 6\log_4 y + 4\log_4 z$

$\log_4 x + \log_4 y^6 + \log_4 z^4$

$= \boxed{\log_4(xy^6z^4)}$

8) $5\log_2 a + 10\log_2 b + 5\log_2 c$

$\log_2 a^5 + \log_2 b^{10} + \log_2 c^5$

$= \boxed{\log_2(a^5b^{10}c^5)}$

Solve each equation.

9) $\log_8 (x-3) - \log_8 10 = 1$

$\log_8\left(\frac{x-3}{10}\right) = 1$

$\frac{x-3}{10} = 8 \cdot 10$

$x - 3 = 80$

$+3 \quad +3$

$\boxed{x = 83}$

10) $\log_4 -2x + \log_4 10 = \log_4 14$

$\log_4(-2x \cdot 10) = \log_4 14$

$-2x(10) = 14$

$\frac{-20x}{-20} = \frac{14}{-20}$

$\boxed{x = -\frac{7}{10}}$

11) $\log_2 2x^2 - \log_2 9 = 5$

$\log_2\left(\frac{2x^2}{9}\right) = 5$

$\frac{2x^2}{9} = 32$

$2x^2 = 288$

$\sqrt{x^2} = \sqrt{144}$

$x = \pm 12$

12) $\ln(x-5) - \ln 9 = 5$

$\ln\left(\frac{x-5}{9}\right) = e^5$

$\frac{x-5}{9} = e^5$

$x - 5 = 9e^5$

$x = 9e^5 + 5$

13) $\log_3 2 + \log_3 2x^2 = 2$

$\log_3(2 \cdot 2x^2) = 2$

$4x^2 = 9$

$\sqrt{x^2} = \sqrt{\frac{9}{4}}$

$x = \pm\frac{3}{2}$

14) $\log_4(x+7) + \log_4 3 = \log_4 45$

$\log_4(x+7)3 = \log_4 45$

$3x + 21 = 45$

$3x = 24$

$x = 8$

15) $\log_6 5x + \log_6 2 = 1$

$\log_6(5x \cdot 2) = 1$

$10x = 6$

$x = \frac{3}{5}$

16) $\log_7(x-5) - \log_7 3 = 2$

$\log_7\left(\frac{x-5}{3}\right) = 2$

$\frac{x-5}{3} = 49$

$x - 5 = 147$

$x = 152$

17) $\log_7(x^2-9) - \log_7 8 = \log_7 27$

$\log_7\left(\frac{x^2-9}{8}\right) = \log_7 27$

$\frac{x^2-9}{8} = 27$

$x^2 - 9 = 216$

$\sqrt{x^2} = \sqrt{225}$

$x = \pm 15$

18) $\log_4 5 + \log_4 4x = 4$

$\log_4(5 \cdot 4x) = 4$

$20x = 256$

$x = \frac{64}{5}$

19) $\ln(x-2) - \ln 6 = 3$

$\ln\left(\frac{x-2}{6}\right) = e^3$

$\frac{x-2}{6} = e^3$

$x - 2 = 6e^3$

$x = 6e^3 + 2$

20) $\log_5 3x - \log_5 6 = 1$

$\log_5\left(\frac{3x}{6}\right) = 1$

$\frac{3x}{6} = 5$

$3x = 30$

$x = 10$

21) $\log_8 3x^2 + \log_8 6 = 3$

$\log_8(3x^2 \cdot 6) = 3$

$18x^2 = 512$

$x^2 = \frac{512}{18}$

$\sqrt{x^2} = \sqrt{\frac{256}{9}}$

$x = \pm\frac{16}{3}$

22) $\log_8 -5x + \log_8 4 = \log_8 44$

$\log_8(-5x \cdot 4) = \log_8 44$

$-20x = 44$

$x = -\frac{11}{5}$

23) $\log_9 3x^2 - \log_9 3 = 3$

$\log_9\left(\frac{3x^2}{3}\right) = 3$

$\frac{3x^2}{3} = 729$

$\sqrt{x^2} = \sqrt{729}$

$x = \pm 27$

24) $\log_4(x+15) + \log_4 x = \log_4 76$

$\log_4(x+15)x = \log_4 76$

$x^2 + 15x = 76$

$x^2 + 15x - 76 = 0$

$(x+19)(x-4) = 0$

$x = 4, -19$

$x = 4$