p495 23-38, 43-52, 60-63, 168 Key

* Questions with * have alternate answers.

(23) $8^x = 40$

$\log_8 8^x = \log_8 40$

$\boxed{x = \log_8 40}$

(24) $5^x = 55$

$\log_5 5^x = \log_5 55$

$\boxed{x = \log_5 55}$

(25) $2.9^{a-4} = 8.1$

$\log_{2.9} 2.9^{a-4} = \log_{2.9} 8.1$

$a - 4 = \log_{2.9} 8.1$

$+4 \quad +4$

$\boxed{a = \log_{2.9} 8.1 + 4}$

* (26) $9^{b-1} = 7^b$

$\log_9 9^{b-1} = \log_9 7^b$

$b - 1 = b \log_9 7$

$-b \quad -b$

$-1 = b \log_9 7 - b$

$-1 = b(\log_9 7 - 1)$

$\log_9 7 - 1 \quad \log_9 7 - 1$

$\boxed{b = \frac{-1}{\log_9 7 - 1}}$

$\log_7 9^{b-1} = \log_7 7^b$

$\log_7 9^{b-1} = b$

$b - 1(\log_7 9) = b$

$b \log_7 9 - \log_7 9 = b$

$-b \log_7 9 \quad -b \log_7 9$

$-\log_7 9 = b - b \log_7 9$

$-\log_7 9 = b(1 - \log_7 9)$

$1 - \log_7 9 \quad 1 - \log_7 9$

$\boxed{b = \frac{-\log_7 9}{1 - \log_7 9}}$

(27) $13^{x^2} = 33.3$

$\log_{13} 13^{x^2} = \log_{13} 33.3$

$\sqrt{x^2} = \sqrt{\log_{13} 33.3}$

$\boxed{x = \pm\sqrt{\log_{13} 33.3}}$

(28) $15^{x^2} = 110$

$\log_{15} 15^{x^2} = \log_{15} 110$

$\sqrt{x^2} = \sqrt{\log_{15} 110}$

$\boxed{x = \pm\sqrt{\log_{15} 110}}$

(29) $6^{3n} > 36$

$6^{3n} > 6^2$

$\frac{3n}{3} > \frac{2}{3}$

$n > \frac{2}{3}$

$\boxed{(\frac{2}{3}, \infty)}$

(30) $2^{4x} \leq 20$

$\log_2 2^{4x} \leq \log_2 20$

$\frac{4x}{4} \leq \frac{\log_2 20}{4}$

$\boxed{x \leq \frac{\log_2 20}{4}}$

*

31 $3^{y-1} \leq 4^y$

$\log_3 3^{y-1} \leq \log_3 4^y$

$y-1 \leq \log_3 4^y$

$y-1 \leq y\log_3 4$

$-y \qquad -y$

$-1 \leq y\log_3 4 - y$

$\frac{-1}{\log_3 4-1} \leq \frac{y(\log_3 4-1)}{\log_3 4-1}$

$\boxed{y \geq \frac{-1}{\log_3 4-1}}$

*

(32) $5^{p-2} \geq 2^p$

$\log_5 5^{p-2} \geq \log_5 2^p$

$p-2 \geq \log_5 2^p$

$p-2 \geq p\log_5 2$

$-p \qquad -p$

$-2 \geq p\log_5 2 - p$

$\frac{-2}{\log_5 2-1} \geq \frac{p(\log_5 2-1)}{\log_5 2-1}$

$\boxed{p \leq \frac{-2}{\log_5 2-1}}$

$\log_2 5^{p-2} \geq \log_2 2^p$

$\log_2 5^{p-2} \geq p$

$p-2(\log_2 5) \geq p$

$p\log_2 5 - 2\log_2 5 \geq p$

$-p\log_2 5 \qquad -p\log_2 5$

$-2\log_2 5 \geq p - p\log_2 5$

$\frac{-2\log_2 5}{1-\log_2 5} \geq \frac{p(1-\log_2 5)}{1-\log_2 5}$

$\boxed{p \leq \frac{-2\log_2 5}{1-\log_2 5}}$

(33) $\log_7 18$

$\boxed{\frac{\log 18}{\log 7}}$

(34) $\log_5 31$

$\boxed{\frac{\log 31}{\log 5}}$

(35) $\log_2 16$

$\boxed{\frac{\log 16}{\log 2}}$

(36) $\log_4 9$

$\boxed{\frac{\log 9}{\log 4}}$

(37) $\log_3 11$

$\boxed{\frac{\log 11}{\log 3}}$

(38) $\log_6 33$

$\boxed{\frac{\log 33}{\log 6}}$

(43) $4^{n+2} = 14.5$

$\log_4 4^{n+2} = \log_4 14.5$

$n+2 = \log_4 14.5$

$-2 \quad -2$

$\boxed{n = \log_4 14.5 - 2}$

(44) $8^{z-4} = 6.3$

$\log_8 8^{z-4} = \log_8 6.3$

$z-4 = \log_8 6.3$

$+4 \quad +4$

$\boxed{z = \log_8 6.3 + 4}$

(45) $7.4^{n-3} = 32.5$

$\log_{7.4} 7.4^{n-3} = \log_{7.4} 32.5$

$n-3 = \log_{7.4} 32.5$

$+3 \quad +3$

$\boxed{n = \log_{7.4} 32.5 + 3}$

(46) $3.1^{y-5} = 9.2$

$\log_{3.1} 3.1^{y-5} = \log_{3.1} 9.2$

$y-5 = \log_{3.1} 9.2$

$+5 \quad +5$

$\boxed{y = \log_{3.1} 9.2 + 5}$

(47) $5^x \geq 42$

$\log_5 5^x \geq \log_5 42$

$\boxed{x \geq \log_5 42}$

(48) $9^{2a} < 120$

$\log_9 9^{2a} < \log_9 120$

$\frac{2a}{2} < \frac{\log_9 120}{2}$

$\boxed{a < \frac{\log_9 120}{2}}$

(49) $3^{4x} \leq 72$

$\log_3 3^{4x} \leq \log_3 72$

$\frac{4x}{4} \leq \frac{\log_3 72}{4}$

$\boxed{x \leq \frac{\log_3 72}{4}}$

* (50) $7^{2n} > 52^{4n+3}$

$\log_7 7^{2n} > \log_7 52^{4n+3}$

$2n > \log_7 52^{4n+3}$

$2n > 4n+3(\log_7 52)$

$2n > 4n \log_7 52 + 3\log_7 52$

$-4n\log_7 52 \quad -4n\log_7 52$

$2n - 4n\log_7 52 > 3\log_7 52$

$\frac{n(2-4\log_7 52)}{2-4\log_7 52} > \frac{3\log_7 52}{2-4\log_7 52}$

$\boxed{n > \frac{3\log_7 52}{2-4\log_7 52}}$

(51) $6^p \le 13^{5-p}$

$\log_{13} 6^p \le \log_{13} 13^{5-p}$

$\log_{13} 6^p \le 5-p$

$p\log_{13} 6 \le 5-p$

$+p \quad +p$

$p\log_{13} 6 + p \le 5$

$p(\log_{13} 6 + 1) \le 5$

$\log_{13} 6 + 1 \qquad \log_{13} 6 + 1$

$$\boxed{p \le \frac{5}{\log_{13} 6 + 1}}$$

(52) $2^{y+3} \ge 8^{3y}$

$\log_2 2^{y+3} \ge \log_2 8^{3y}$

$y+3 \ge \log_2 8^{3y}$

$y+3 \ge 3y\log_2 8$

$-y \quad -y$

$3 \ge 3y\log_2 8 - y$

$3 \ge y(3\log_2 8 - 1)$

$3\log_2 8 - 1 \qquad 3\log_2 8 - 1$

$$\boxed{y \le \frac{3}{3\log_2 8 - 1}}$$

(60) $10^{x^2} = 60$

$\log_{10} 10^{x^2} = \log_{10} 60$

$\sqrt{x^2} = \sqrt{\log_{10} 60}$

$$\boxed{x = \pm\sqrt{\log_{10} 60}}$$

(61) $4^{x^2-3} = 16$

$4^{x^2-3} = 4^2$

$x^2 - 3 = 2$

$+3 \quad +3$

$\sqrt{x^2} = \sqrt{5}$

$$\boxed{x = \pm\sqrt{5}}$$

$\log_4 4^{x^2-3} = \log_4 16$

$x^2 - 3 = \log_4 16$

$+3 \qquad +3$

$\sqrt{x^2} = \sqrt{\log_4 16 + 3}$

$x = \pm\sqrt{\log_4 16 + 3}$

(62) $9^{6y-2} = 3^{3y+1}$

$3^{2(6y-2)} = 3^{3y+1}$

$12y - 4 = 3y + 1$

$9y - 4 = 1$

$9y = 5$

$$\boxed{y = \frac{5}{9}}$$

(63) $8^{2x-4} = 4^{x+1}$

$2^{3(2x-4)} = 2^{2(x+1)}$

$6x - 12 = 2x + 2$

$4x - 12 = 2$

$4x = 14$

$$\boxed{x = \frac{7}{2}}$$

(68) Rosamaria

* Alternates for 50, 51, 52

(50) $7^{2n} > 52^{4n+3}$

$\log_{52} 7^{2n} > \log_{52} 52^{4n+3}$

$\log_{52} 7^{2n} > 4n+3$

$2n \log_{52} 7 > 4n+3$

$-4n \quad -4n$

$2n \log_{52} 7 - 4n > 3$

$\frac{n(2\log_{52} 7 - 4)}{2\log_{52} 7 - 4} > \frac{3}{2\log_{52} 7 - 4}$

$$n > \frac{3}{2\log_{52} 7 - 4}$$

(51) $6^p \leq 13^{5-p}$

$\log_6 6^p \leq \log_6 13^{5-p}$

$p \leq \log_6 13^{5-p}$

$p \leq 5 - p(\log_6 13)$

$p \leq 5\log_6 13 - p\log_6 13$

$+p\log_6 13 \quad +p\log_6 13$

$p + p\log_6 13 \leq 5\log_6 13$

$\frac{p(1+\log_6 13)}{1+\log_6 13} \leq \frac{5\log_6 13}{1+\log_6 13}$

$$p \leq \frac{5\log_6 13}{1+\log_6 13}$$

(52) $2^{y+3} \geq 8^{3y}$

$\log_8 2^{y+3} \geq \log_8 8^{3y}$

$\log_8 2^{y+3} \geq 3y$

$y + 3(\log_8 2) \geq 3y$

$y\log_8 2 + 3\log_8 2 \geq 3y$

$-y\log_8 2 \quad -y\log_8 2$

$3\log_8 2 \geq 3y - y\log_8 2$

$\frac{3\log_8 2}{3-\log_8 2} \geq \frac{y(3-\log_8 2)}{3-\log_8 2}$

$$y \leq \frac{3\log_8 2}{3-\log_8 2}$$