

Properties of Rational Exponents

Definitions:

1. Product of Power Property - $a^m \cdot a^n = a^{m+n}$

Ex. $3^{\frac{1}{2}} \cdot 3^{\frac{3}{2}} = 3^{\frac{4}{2}} = 3^2$

2. Power of a Power Property - $(a^m)^n = a^{mn}$

Ex. $(4^{\frac{3}{2}})^2 = 4^{\frac{6}{2}} = 4^3$

3. Power of a Product Property - $(ab)^m = a^m b^m$

Ex. $(9 \cdot 4)^{\frac{1}{2}} = 9^{\frac{1}{2}} \cdot 4^{\frac{1}{2}}$

4. Negative Exponent Property - $a^{-m} = \frac{1}{a^m}$

Ex. $25^{-\frac{1}{2}} = \frac{1}{25^{\frac{1}{2}}}$

5. Quotient of Powers Property - $\frac{a^m}{a^n} = a^{m-n}$

Ex. $\frac{6^{\frac{5}{2}}}{6^{\frac{1}{2}}} = 6^{\frac{4}{2}} = 6^2$

6. Power of a Quotient Property - $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Ex. $\left(\frac{8}{27}\right)^{\frac{1}{3}} = \frac{8^{\frac{1}{3}}}{27^{\frac{1}{3}}}$

Example 1.) Simplify the expression.

a.) $5^{\frac{1}{2}} \cdot 4^{\frac{1}{4}} = 5^{\frac{3}{4}}$

b.) $(8^{\frac{1}{2}} \cdot 5^{\frac{1}{3}})^2 = 8^{\frac{2}{2}} \cdot 5^{\frac{2}{3}} = 8 \cdot 5^{\frac{2}{3}}$

c.) $(2^4 \cdot 3^4)^{-\frac{1}{4}} = \frac{2^{-4} \cdot 3^{-4}}{1} = \frac{1}{2^4 \cdot 3^4} = \frac{1}{6}$

d.) $\left(\frac{12^{\frac{1}{3}}}{4^{\frac{1}{3}}}\right)^2 = \frac{12^{\frac{2}{3}}}{4^{\frac{2}{3}}} = 3^{\frac{2}{3}}$

e.) $\frac{7^{\frac{1}{3}}}{7^{\frac{1}{3}}} = 7^{\frac{2}{3}}$