Radical Eq & Inequalities

① $(\sqrt{a-5})^2 = (a-7)^2$

$a-5 = a^2-14a+49$

$-a \quad +5 \quad -a \quad +5$

$a^2-15a+54=0$

$(a-9)(a-6)=0$

$a=9, 6$

$\sqrt{9-5} = 9-7$

$2=2$ ✓

$\sqrt{6-5} = 6-7$

$1=-1$ ×

$\boxed{a=9}$

② $1 = \sqrt{3x-8} - \sqrt{5-x}$

$(1+\sqrt{5-x})^2 = (\sqrt{3x-8})^2$

$(1+\sqrt{5-x})(1+\sqrt{5-x}) = 3x-8$

$1+\sqrt{5-x}+\sqrt{5-x}+5-x = 3x-8$

$6-x+2\sqrt{5-x} = 3x-8$

$-6 \quad +x \qquad +x \quad -6$

$\frac{2\sqrt{5-x}}{2} = \frac{4x-14}{2}$

$(\sqrt{5-x})^2 = (2x-7)^2$

$5-x = 4x^2-28x+49$

$-5 \quad +x \qquad +1x \quad -5$

$4x^2-27x+44=0$

16 11

$4x^2-16x-11x+44=0$

$4x(x-4)-11(x-4)$

$(x-4)(4x-11)=0$

$x=4, \frac{11}{4}$

$x=4$

③ $y - \sqrt{2y+1} < 3$

$-6 \quad -6$

$-\sqrt{2y+1} < -3$

$-1 \quad -1$

$(\sqrt{2y+1})^2 > 3^2$

$2y+1 > 9$

$2y > 8$

$\boxed{y > 4}$

$2y+1 \geq 0$

$2y \geq -1$

$y \geq -\frac{1}{2}$

④ $\sqrt{3x+6} + 2 \leq 5$

$-2 \quad -2$

$(\sqrt{3x+6})^2 \leq 3^2$

$3x+6 \leq 9$

$-6 \quad -6$

$3x \leq 3$

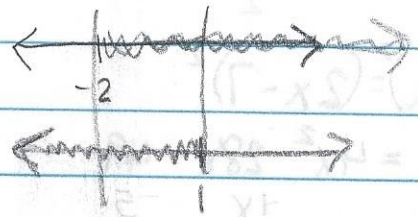
$x \leq 1$

$\boxed{-2 \leq x \leq 1}$

$3x+6 \geq 0$

$3x \geq -6$

$x \geq -2$



⑤ $2+\sqrt{4y-4} \le 6$

$-2 \quad -2$

$(\sqrt{4y-4})^2 \le 4^2$

$4y-4 \le 16$

$4y \le 20$

$y \le 5$

$4y-4 \ge 0$

$4y \ge 4$

$y \ge 1$

$\boxed{1 \le y \le 5}$

⑥ $1+\sqrt{7x-3} > 3$

$-1 \quad -1$

$(\sqrt{7x-3})^2 > 2^2$

$7x-3 > 4$

$7x > 7$

$\boxed{x > 1}$

$7x-3 \ge 0$

$7x \ge 3$

$x \ge \frac{3}{7}$