

Section 5.5B – Apply the Factor Theorem

Recall:

Factor - a number or quantity that when multiplied with another produces a given number or expression
 Ex. $2 \times 6 = 12$ hence, 2 and 6 are factors of 12

AC Method – for factoring quadratic expressions

What are the two methods for dividing polynomials?

- 1.) Polynomial Long Division
- 2.) Synthetic Division

Note – this can only be used when the divisor is a binomial

What does it mean to have a remainder of zero?

$$\frac{6}{3} = 2 \quad \frac{100}{5} = 20$$

Answer: The divisor is a factor of the dividend.

Definition:

Factor Theorem – a polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$
 Thus, if synthetic division yields a remainder of zero then $x - k$ is a factor

Example 1.) Factor $f(x) = 3x^3 - 4x^2 - 28x - 16$ completely given that $(x + 2)$ is a factor. $(x+2)=0$
 $-2-2$

Compare to knowing that 7 is a prime factor of 56 and wanting to find its other prime factors. $x=-2$
 Do prime factorization with 56.

$$\begin{array}{r|rrrr}
 -2 & 3 & -4 & -28 & -16 \\
 & \downarrow & & & \\
 \hline
 x \rightarrow & 3x^2 & -10x & -8 & 0
 \end{array}$$

$$\begin{aligned}
 & (x+2)(3x^2 - 10x - 8) \qquad \begin{array}{l} -24 \\ -12 + 2 \end{array} \\
 & \quad \quad \quad \underbrace{3x^2 - 12x + 2x - 8} \\
 & \quad \quad \quad 3x(x-4) \quad 2(x-4)
 \end{aligned}$$

$$f(x) = (x+2)(3x+2)(x-4)$$

Recall:

Zero Product Property – if $ab = 0$ then $a = 0$ or $b = 0$

Example 2.) One zero of $f(x) = x^3 - 2x^2 - 23x + 60$ is $x = 3$. What are all the zeros of ?

Note – this means $(x - 3)$ is a factor of f

$$\begin{array}{r|rrrr} 3 & 1 & -2 & -23 & 60 \\ & \downarrow & & & \\ x & 1x^2 & 1x & -20 & 0 \end{array}$$

$$(x-3)(x^2 + x - 20)$$

$$\boxed{f(x) = (x-3)(x+5)(x-4)}$$
$$\boxed{x = 3, -5, 4}$$

$$\begin{array}{l} x-3=0 \\ x+5=0 \\ x-4=0 \\ -20 \\ 5 \quad -4 \end{array}$$

Recall:

Quadratic Formula – can be used to solve any quadratic equation

Example 3.) One solution of $g(x) = x^3 + 2x^2 - 9x - 18$ is $x = -2$. Find the other solutions.

Note – this means that $x + 2$ is a factor of g

$$\begin{array}{r|rrrr} -2 & 1 & 2 & -9 & -18 \\ & \downarrow & & & \\ x & 1x^2 & 0x & -9 & 0 \end{array}$$

$$(x+2)(x^2 - 9)$$

$$\boxed{g(x) = (x+2)(x+3)(x-3)}$$
$$\boxed{x = -2, 3, -3}$$

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