Solving Log Equations and Inequalities

Definition:

Logarithmic Equations – these are equations that involve logarithms of variable expressions

Property of Equality for Logarithmic Equations – if b, x, and y are positive numbers, $log_b x = log_b y$ if and only if x = y

Ex. If $log_2 x = log_2 7$, then $x = 7$

Example 1) Solve $log_5(4x - 7) = log_5(x + 5)$

$4x - 7 = x + 5$

$-x \quad -x$

$3x - 7 = 5$

$+7 \quad +7$

$\frac{3x}{3} = \frac{12}{3}$

$x = 4$

* This is the key!

The property of equality for exponential equations implies that if x = y, then you can exponentiate each side and obtain the equation $b^x = b^y$. This technique can be used to solve some logarithmic equations.

Example 2) Solve $log_4(5x - 1) = 3$

$5x - 1 = 4^3$

$5x - 1 = 64$

$\frac{5x}{5} = \frac{65}{5}$

$x = 13$

Logarithmic Inequalities – an inequality that involves logarithms.

Property of Inequality for Logarithmic Functions – If b > 1, x > 0, and $\log_b x > y$, then $x > b^y$. If b > 1, x > 0, and $\log_b x < y$, then $0 < x < b^y$.

- So if the inequality is > or ≥, then that will be your answer.
- If the inequality is < or ≤, then you have to find the lower boundary because it cannot be ≤ 0.

Examples) $\log_3 x > 4$

$x > 3^4$

$x > 81$

$(81, \infty)$

$\log_2 x < 4$

$x < 16$

$0 < x < 16$

$(0, 16)$

$\log_4(x+3) > \log_4(2x+1)$

$x+3 > 2x+1$

$-x+3 > 1$

$-x > -2$

$x < 2$

$x+3>0$ $2x+1>0$

$x>-3$ $2x>-1$

$x>-\frac{1}{2}$

$-\frac{1}{2} < x < 2$

$\left(-\frac{1}{2}, 2\right)$

$\log_5(2x+1) \leq \log_5(x+4)$

$2x+1 \leq x+4$

$x+1 \leq 4$

$x \leq 3$

$2x+1>0$ $x+4>0$

$x>-\frac{1}{2}$ $x>-4$

$-\frac{1}{2} < x \leq 3$

$\left(-\frac{1}{2}, 3\right]$

$\log_7(2x+8) > \log_7(x+5)$

$2x+8 > x+5$

$x+8 > 5$

$x > -3$

$(-3, \infty)$

$\log_3(7x-6) < \log_3(4x+9)$

$7x-6 < 4x+9$

$3x-6 < 9$

$3x < 15$

$x < 5$

$7x-6>0$ $4x+9>0$

$x>\frac{6}{7}$ $x>-\frac{9}{4}$

$\frac{6}{7} < x < 5$

$\left(\frac{6}{7}, 5\right)$

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23-33 ODD,
38